FIRST STEPS
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**THE ANALOG THING – First Steps**  
First English edition, version 1.3

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1. WELCOME

Computing today is a digital monoculture. At anabrid, we want to see a comeback of analog computing and, eventually, an analog-digital hybrid computing future, because:

- Analog computing is vastly more energy-efficient than digital computing [1], promising to lower our computational CO₂ footprint and energy costs significantly.
- As the exponential growth of digital computing power described by Moore’s Law approaches its limits [2], analog-digital hybrid technology promises to sustain growth in computer performance for decades to come.
- Analog computing may protect critical infrastructure by reducing attack surfaces of online operational technology and by offering failover options. [3]
- Analog computing is a fascinating intellectual and artistic activity. As a computing paradigm it is radically different from Turing machine-type thinking, offering an excellent way to learn about mathematics, science, and engineering. [4] It also enables creative expression when used, for instance, in conjunction with voltage-controlled audio synthesizers.

You are invited to join the quest to bring analog computing back.

THE ANALOG THING (or simply THAT) is here to get you started. THAT is a high-quality, low-cost, open-source, not-for-profit cutting-edge analog computer. It computes with continuous voltages rather than with zeroes and ones. Capable of solving (sets of) differential equations, it enables the modeling of a broad range of dynamic systems. This booklet offers guidance to users taking first steps with THAT, focusing on the qualitative rather than quantitative aspects of analog computing. We hope you find this booklet, along with THAT, its online resources, and its community of users enjoyable resources in your learning inquiries and in our shared quest to make computing more diverse!


2. REQUIREMENTS

To get started with THAT, the following items are required:

- a system to display the output of THAT (time-varying voltages). See Section 7 of this guide for options.
- a USB power supply such as a phone charger with a USB-C plug (not included)
- suitable cables and adapters to connect THAT to the output display setup used (not included)
- a set of 2 mm banana plug patch cables (included)

3. WHAT IS IN THE BOX?

THAT ships in a box that contains:

- 1 power cord: USB-A to USB-C
- 1 stereo RCA-to-RCA cable
- 1 set (30 pcs.) 2 mm banana plug patch cables
- 1 master-to-minion ribbon cable
- The First Steps booklet

The box does not include:

- an oscilloscope or other visual display system
- a USB power supply (many people have spare ones)
- BNC adapters/cables to connect THAT to an oscilloscope

For a more comprehensive details and documentation of THAT, please check https://the-analog-thing.org/docs.
For accessories, please check the THAT shop at https://shop.anabrid.com.
Analog computing is one of the three computational paradigms: analog, digital, and quantum computing. Each of these paradigms has its own strengths and weaknesses, and they differ in both their development and prevalence. Digital computing is ubiquitous today, while quantum computing is in its infancy. Analog computing is tried and tested, yet has fallen out of fashion and is largely unknown today. THAT is here to change this because analog computing offers exceptional power efficiency and performance, both of which are much needed as the limits of Moore's Law are approaching.

Generally speaking, analog computing is about modeling dynamic systems, i.e., systems that change over time according to known relationships. Examples include market economies, the spread and control of diseases, population dynamics, nutrient absorption, nuclear chain reactions, and mechanical systems. Models of dynamic systems are useful similarly to how architectural models are useful in building design and crash test dummies are useful in car safety engineering. They offer insights into matters that would be too difficult, laborious, expensive, or harmful to study directly. Analog computing can serve a variety of purposes. It may help understand what is (models of), or it may help bring about what should be (models for). It may be used to explain in educational settings, to imitate in gaming, to predict in the natural sciences, and to control in engineering – or it may be pursued for the pure joy of it. Analog computing is also a great way to learn about calculus, science, and engineering.

Analog computers are modular and analog computer “programming” is a process of translating the behavior of a given system into patched connections between computing elements – the modules that make up an analog computer. As intermediate steps, this process requires that temporal behavior be described mathematically in the form of differential equations and, in turn, that these equations be converted into patch diagrams. While solutions of algebraic equations are single values, solutions of differential equations are functions – i.e., relationships that can be presented as graphs. Consequently, analog computers produce output in the form of (typically two-dimensional) graphs. All differential equations can be modeled with just a few kinds of computing elements: inverters, summers, multipliers, and, crucially, integrators.

Analog computing has one of its major roots in the invention of the mechanical “disk-globe-and-cylinder” integrator. Comparable to how interlocked, different-sized gears multiply rotation rates and can thus be seen as performing mathematical multiplication, the disk-globe-and-cylinder integrator can be seen as performing mathematical integration by computing the area under a curve in x/y space. The disk-globe-and-cylinder, shown below, consists of a disk, a sphere (or globe), and a cylinder, each mounted to rotate.

Disk, sphere, and cylinder are configured with the circular surface of the disk touching the sphere and the sphere, in turn, touching the curved surface of the cylinder such that rotating either one of the three will, under most circumstances, turn the other two. In addition to its ability to rotate, the sphere can also be moved linearly back and forth along its rotational axis, which runs parallel to the disk's diameter, without ever losing contact with either disk or cylinder. With the disk rotating at a constant rate \( a \) to mark the passage of time, the input rotation \( b \) is made to follow a curve that changes over its horizontal time axis, moving the sphere across the diameter of the disk. The resulting output rotation \( c \) of the cylinder is then the integral of the area under curve \( b \).

By feeding the output rotation of the cylinder \( c \) back to drive the input rotation \( b \), the integrator can be used, in principle, to solve first-order differential equations and plot their solution graphs. Two integrators in series with the output of the second integrator fed back to drive the input of the first one allows a second-order differential equation to be solved, and so on. This working principle of mechanical analog computers can be implemented more efficiently using analog electronics, specifically by patching operational amplifiers such that they integrate their electrical inputs over time by charging and discharging capacitors.
5. USING THAT

THAT is an electronic analog computer designed for desktop use to solve (sets of) differential equations. With its patch panel instead of keyboard, mouse, and monitor, its user interface differs noticeably from those of its digital stored-program cousins. The patch panel is divided into groups of analog computing elements such as integrators, summers, and multipliers. These are listed and explained in Section 6. Each computing element on THAT has one or more inputs and an output, accessible via jacks on the plug board. Input jacks are marked with circles, and output jacks are marked with triangles.

THAT is patched by connecting computing elements into feedback circuits via patch cables – single-wire cables with banana plugs on both ends. Note that patch cable plugs do not insert completely into THAT’s patch panel. Multiple patch cable plugs may be stacked in order to connect multiple cables to the same jack.

The output of each computing element can be fed to one or more inputs of other elements. However, an input of a computing element may only take a single signal, such as an output of another computing element. To solve differential equations, chains of connected computing elements must contain at least one closed feedback loop. Feedback around one integrator allows first-order differential equations to be solved, feedback around two integrators allows second-order differential equations to be solved, and so on.

THAT represents quantities as time-varying voltages in a range with fixed boundaries. This range, called the *machine unit*, is thought of as −1 to 1. The actual voltage range THAT uses is −10 V to +10 V. Translating patterns of change in dynamic systems into differential equations and further into analog computer patches commonly involves the scaling of speed and quantities.

Scaling speed allows users to compress or stretch the independent variable time, typically by several orders of magnitude, to fit patch runtimes into convenient time frames for observation and measurement. In this way, the instantaneous decay of a volatile compound can be simulated slowly enough for observation and interactive manipulation, while population dynamics occurring over decades or centuries can be simulated in the blink of an eye. Scaling quantities is about multiplying all values occurring in a given patch uniformly, such that the greatest value occurring in the patch fits into and makes good use of the machine unit interval. A model of the global human population size, for example, representing millions of individuals, would not involve millions of volts but be scaled to fit within −1 to +1 (i.e., −10 V to +10V).

THAT outputs solutions of differential equations in the form of time-varying voltages. These can be captured by an attached digital computer for further processing, or they can be studied visually and interactively using an oscilloscope or a similar display system. Just as single-board digital computers allow “headless” operation without a monitor, THAT can operate without a visual display. Often, however, and especially when taking first steps with THAT, using a visual display system is advisable. Section 7 shows several options.

Once THAT is connected to a power supply and a display system, it is ready to be patched. This typically begins with an understanding of the quantitative relationships that are present in some dynamic system, expressed in the form of one or more differential equations. This then guides the development of a patching diagram that can then be implemented on THAT’s patch panel. Quantities enter the patch via coefficient potentiometers. These can be set at the beginning of the patching process by connecting the input of each potentiometer used to +1, setting the mode selector to COEFF, selecting the potentiometer using the coefficient selector knob, monitoring its output on the panel meter, and turning the potentiometer knob until the desired value is shown.

Setting THAT to OP, REP, or REPF mode will then start the patch to compute the unknown solution(s) of the differential equation(s) and output them as time-varying voltages that correspond to the temporal change in the modeled dynamic system. The most immediate way to read these solutions and manipulate them interactively is to visualize them live on an oscilloscope or a similar setup.
6. VISUAL DISPLAY SYSTEM SETUP

Patches running on THAT produce output in the form of time-varying voltages at various places on the patch panel. The most immediate way to read these solutions and manipulate them interactively is to visualize them on an oscilloscope or a similar visual display system. Up to four values can be selected and connected to a visual display system.

This section describes several ways to set up THAT with a visual display system. These vary considerably in terms of capabilities and affordability. Ideally, visual display systems used with THAT should be capable of visualizing multiple (two or ideally four) channels, frequencies of 200kHz, and support x/y display mode. Less capable but more affordable systems such as software oscilloscopes with a single channel and a 20kHz frequency limit offer less than ideal yet satisfactory performance for the applications covered in this guide.

6.1 OSCILLOSCOPE

THAT can be used with various kinds of oscilloscopes, such as conventional cathode ray tube oscilloscopes, digital oscilloscopes, and USB oscilloscopes in conjunction with PCs. Connect one or more of the outputs X, Y, Z, and U on the back of THAT directly to the inputs of the oscilloscope. Use suitable cables such as the included RCA-to-RCA cable in combination with RCA-to-BNC adapters (not included). Up to four channels can be connected if supported by the oscilloscope. Refer to your oscilloscope manual for setting the display mode, channel sensitivity, and time deflection. Connect the USB-C IN on the back of THAT to a USB power supply using a USB-C cable.

Using a desktop oscilloscope

Using a PC with an external USB oscilloscope
6.2 PC WITH AUDIO INPUT INTERFACE

THAT can be used in conjunction with software oscilloscopes running on PCs and taking input via audio input interfaces, such as external USB audio interfaces and standard sound cards. Connect one or two of the outputs X, Y, Z, and U on the back of THAT to the RCA audio inputs of the audio interface using suitable cables and adapters. The RCA-to-RCA cable included with THAT is suitable for connections to USB audio interfaces with RCA jacks. Soundcards with a 3.5 mm jack require cables or adapters that are not included with THAT. The number of audio input channels available (typically two: one left and one right) determines the number of values that can be monitored. Install an oscilloscope software application on the digital computer and select the audio input interface as its input device. Connect the USB-C IN on the back of THAT to a USB power supply using a USB-C cable.

Audio input interfaces contain capacitors that alter signal waveforms by filtering out DC and low frequency components. Note the difference between the original waveform and the waveform measured using a sound card and software oscilloscope shown here:

![Original and measured waveforms](image)

As qualitative demonstrations, the applications discussed in this guide are not seriously affected by this.

Using a PC with an external USB audio interface

Using a PC with a soundcard interface
7. FEATURES OF THAT

1. USB-C Power In  
2. MINION IN Port  
3. RCA Out  
4. RCA Trigger Out  
5. HYBRID Port  
6. MASTER OUT Port

7. Integrators  
8. Inverters  
9. Resistor Networks  
10. Diodes  
11. Output Jacks  
12. Panel Meter  
13. State LEDs  
14. Mode Selector

15. OP-Time Potentiometer  
16. Coefficient Selector  
17. Coefficient Potentiometers  
18. Capacitors  
19. Machine Units  
20. Multipliers  
21. Comparators  
22. Summers  
23. Coefficients
1. **USB-C Power In.** Power input jack (the USB data pins are not used). Use a USB power supply unit such as a phone charger or any other USB outlet with a USB-C cable.

2. **MINION IN Port.** When applications require more computing elements than are available on a single THAT, multiple THATs can be linked in a “minion chain” using their “MASTER OUT” and “MINION IN” ports. Connecting the MASTER OUT port of a THAT to the MINION IN port of another THAT with the included ribbon cable makes the first THAT the “master” and the second THAT its “minion.” There is no limit to the number of THATs that can be linked in a minion chain.

3. **RCA Out.** The four RCA jacks labeled X, Y, Z, and U provide the signals plugged to the x, y, z, and u jacks on the patch field, attenuated to a ±1 V range, which is compatible with audio hardware such as sound cards, allowing software oscilloscopes to be used.

4. **RCA Trigger Out** provides a trigger signal for oscilloscopes when THAT is used in REP (repeat) or REPF (repeat fast) mode.

5. **HYBRID Port** allows THAT to be controlled from digital devices, enabling the development of analog-digital hybrid programs. Outputs on the HYBRID Port are attenuated and shifted to 0 V to 3.3 V to facilitate analog/digital conversion.

6. **MASTER OUT Port.** See 2. MINION IN Port.

7. **Integrators** integrate the sum of their (weighted) input values over time. Each integrator has five inputs (two of which are weighted by factor 10) and one output via two output jacks. The IC (initial condition) input allows the start value of an integration to be set. Integrators change the sign of their output values implicitly.

8. **Inverters** yield the input value with the opposite sign.

9. **Resistor Networks** can be used to add inputs to an integrator, summer, or inverter by connecting the Sj jack of a resistor network with the Sj jack of the computing element.

10. **Diodes** support various applications.

11. **Output Jacks.** Patch cable connections from any output on the patch panel into these jacks make those outputs available at the RCA Out ports as well as via the HYBRID port at the back of THAT, shifted to a narrower voltage range (see 3. RCA Out above).

12. **Panel Meter** displays values or operation times, depending on the Mode Selector position. In COEFF mode, the output value of the coefficient potentiometer selected using the Coefficient Selector is shown in the unit range 0 to 1. In the MINION, IC, OP, or HALT modes, the value patched into the U OUTPUT jack is shown in the unit range –1 to 1. In REP mode, the OP TIME is shown in the 0 to 10 sec range. In REPF mode, the OP TIME is shown in the 0 to 100 ms range.

13. **State LEDs** display device states: OP=patch running, IC=initial condition mode, OL=overload.

14. **Mode Selector** controls the operational mode of THAT: In COEFF mode, the value of the coefficient selected using the COEFFICIENT selector is displayed on the Panel Meter. In IC mode, the outputs of the integrators are set to the values applied to their respective IC (initial condition) inputs (with the opposite sign) to set the stage for a patch run. In OP (operate) mode, the current patch is run. In HALT mode, integration is suspended and outputs are held at their last values. In REP (repeat) mode, THAT repeatedly follows a process of briefly entering IC mode, then OP mode. This allows steady graphs to be displayed on an oscilloscope. REPF: As REP mode, but 100 times faster. In MINION mode, THAT is controlled by another THAT that operates as the MASTER.

15. **OP-Time Potentiometer** sets time spent in OP mode between 0 and 10 seconds in REP (repeat) mode and between 0 to 100 milliseconds in REPF (repeat fast) mode.

16. **Coefficient Selector** selects the coefficient potentiometer value displayed on Panel Meter in COEFF mode.

17. **Coefficient Potentiometers** are used to change the values of coefficients (see 23. Coefficients).

18. **Capacitors** support various applications.

19. **Machine Units** provide –1 and +1 unit values.

20. **Multipliers** multiply values supplied to their inputs.

21. **Comparators** allow conditional switching. If A+B>0, then the input to > is available at the two output jacks, otherwise the input to < is available at the two output jacks.

22. **Summers** add up values supplied to their inputs. Each summer has seven inputs (three of which are weighted by factor 10) and one output via two output jacks. Summers invert the sign of their output values implicitly.

23. **Coefficients.** Input and output jacks of the Coefficient Potentiometers.
8. APPLICATIONS

This section presents six analog computer applications from several fields, exemplifying typical modeling workflows from the translation of differential equations into patch diagrams to patch panel connections and parametric exploration.

8.1 RADIOACTIVE DECAY

A minimal analog computer patch consists of an integrator whose output is fed back as its input. Imagine rotation $c$ in the image in Section 4 being fed back to drive rotation $b$. Depending on the rotational direction of this feedback, $c$ plotted over $a$ results in an either exponentially growing or exponentially decaying curve and thus offers a model for phenomena with exponential dynamics. Let’s give this a try electronically by setting up an analog computer patch to model the decay in a radioactive sample. Exponential decay can be described with the first-order differential equation shown here.

In this equation, $\lambda$ denotes the decay coefficient. Note that $\dot{N}$ in Newton’s notation is the equivalent of $dN/dt$ in Leibniz’s notation. To translate this equation into a patching diagram, the Kelvin feedback technique can be used. As a first step, this technique typically requires arranging the equation(s) such that the highest derivative is isolated on the left side of the equal sign. Showing $\dot{N}$ isolated on is left, the equation above is already arranged in this way. Establishing the feedback loop required to solve the equation can now take advantage of the equality of both sides of the equation. For this purpose, the highest derivative on the left of the equal sign is, for the time being, assumed to be known. The term on the right of the equal sign $-N\lambda$ is then modeled with a suitable chain of computing elements. In the radioactive decay example, this is simply an integrator connected to a potentiometer that implements the decay coefficient. The output of this chain of computing elements, known to be equal to the highest derivative, is then connected to the input of the first computing element in the chain, closing the feedback loop. In the radioactive decay example, a second potentiometer is needed to set the size of the sample at the start of the patch run (i.e., the initial condition of the integrator).

As integrators change the signs of their output, feeding an integrator’s output back to its input is sufficient to model decay (i.e., negative exponential growth). To display diminishing positive values, however, the initial value of the integrator’s output must be negative, and the sign of the output must be changed using an inverter. Two coefficient potentiometers allow the initial number of molecules $N_0$ and the decay coefficient $\lambda$ to be adjusted.

To run this patch, put THAT in COEFF mode. Plug cable connections on the patch panel as shown in the patch diagram. Use the coefficient selector to select coefficient potentiometers 1 and 2 in turn and set both to 0.5. Connect your display system to RCA Out X on the back of THAT. Run the patch in REP (repeat) mode to display a flicker-free image on the display system. As the patch runs, change coefficients 1 and 2 and observe how the graph changes. Also, adjust the OP Time and run the patch in REPF mode to observe how these changes affect the output.
According to Newton’s second law of motion, \( F_m = m \times a \). The force with which the damper resists movement \( F_D \) is a damping coefficient \( D \) times the speed \( v \) of its vertical displacement. The force exerted by the spring \( F_s \) is a spring coefficient \( s \) times its vertical displacement \( y \). The speed \( v \) is the first derivative of vertical displacement over time, which we denote by \( y \), and the acceleration \( a \) is the second derivative of vertical displacement over time, which we denote by \( y \).

This yields \( m \ddot{y} + D \dot{y} + sy = 0 \) or, resolved for the highest derivative \( \ddot{y} \):

\[
\ddot{y} = \frac{1}{m}(-DY + sy)
\]

Developing a patching diagram from this second-order differential equation takes advantage of the equality of both sides of the equal sign. Assuming that \( \ddot{y} \) is known, we model the term on the right of the equal sign using two integrators and feed the resulting lower derivatives, with coefficients applied and summed, back to the input of the first integrator, as shown in the diagram on the top right.

Run the patch in REPF (repeat fast) mode at 80 ms OP-time to view a flicker-free image on the display system. As the patch runs, change the settings of coefficient potentiometers 1 through 4 and observe the suspension dynamics change. This patch also applies to damped oscillators in scenarios other than vehicle suspension tuning, for example in earthquake safety engineering and electronic circuit design.
8.3 LUNAR LANDING

“Houston, Tranquility Base here. The Eagle has landed.” Neil Armstrong radioed to Houston Mission Control after landing the Apollo 11 Lunar Module on the surface of the Moon on July 20, 1969. Spacecraft Communicator Charles Duke replied from Houston, “Roger, Tranquility. We copy you on the ground. You got a bunch of guys about to turn blue. We’re breathing again. Thanks a lot!” Commander Neil Armstrong flew the lunar landing approach in tight interaction with pilot Buzz Aldrin and the Lunar Module Guidance Computer. Descending from a lunar orbit, the lander moved on an initially near-horizontal but soon increasingly steep downward trajectory. As lunar surface features came into view ever more clearly from lower and lower altitudes, Armstrong repeatedly used a joystick to redesignate the targeted landing site, aiming for terrain features suitable for a safe landing.

Meanwhile, the computer ensured a steady descent rate, which Armstrong could adjust using an (increase)-neutral-(decrease) toggle switch. Controlling the rate of descent in this way, Armstrong was able to trade fuel for time during which safer landing sites could be spotted and reached. In doing so, he had to use the fuel conservatively, to ensure a safe touchdown. After Armstrong decided to fly beyond a boulder field, the lander finally touched down with less than 5 percent fuel left in its descent stage. To make matters worse, sloshing motions of the fuel inside the tanks caused false, even lower fuel level readings, and the computer threw up a total of five false program alarms during the landing approach, turning the first manned lunar landing into one of the most hair-raising maneuvers of NASA’s lunar program.

The analog computer patch shown here simulates the powered descent of a lunar lander during its final approach, putting you in charge of the descent engine throttle. A system of three differential equations models vertical velocity \( v \) under the influence of lunar gravity \( g \), altitude \( h \), and fuel level \( F \) remaining from a limited initial supply. Coefficient potentiometer 4 is your descent engine throttle.

Set coefficient potentiometers 1, 2, 3 and 5 to the values shown in the patching diagram below. Connect \( h \) to OUT X and connect \( F \) to OUT U. Connect your oscilloscope or other display system to RCA Out X and U on the back of THAT. Set the display system to roll mode. Run the patch in THAT’s OP mode and use coefficient potentiometer 4 to control your descent engine’s burn rate and thrust. Watch your altitude and remaining fuel level on the display system and monitor your remaining fuel level, also on the panel meter. Good luck and Godspeed!

\[
\begin{align*}
v(t) &= -\int (g - T) \, dt \\
F(t) &= F(0) - \int \alpha T \, dt \\
h(t) &= h(0) - \int v(t) \, dt
\end{align*}
\]
Neurons in the central nervous system receive nerve impulses from other “upstream” neurons and, if such inputs exceed their firing thresholds, give off nerve impulses to “downstream” neurons. Computationally speaking, the aggregation of incoming nerve impulses (also referred to as summation) is a process of integration, albeit within a limited time window, such that incoming impulses soon expire, and only the most recent ones carry weight in the integration as time passes. A few sporadic impulses are usually not enough to exceed the firing threshold. Instead, it takes either multiple upstream neurons to fire together at least near-simultaneously, or a single upstream neuron to send multiple impulses in short bursts. James L. Hindmarsh and R. Malcolm Rose proposed a model for this neuronal bursting in 1984. The model, consisting of the three first-order differential equations shown below, responds to inputs to $I_{ext}$. The variables $x$, $y$, and $z$ correspond to the neuron’s (bursting) output potential, the transport of sodium and potassium through fast ion channels, and the transport of other ions through slow channels, respectively.

To add a 10-weighted input to the first (red) integrator, a resistor network is connected to its summing junction (SJ) jack. To operate the second (blue) integrator more slowly than the other two, its output is patched to its SLOW jack. Set the coefficient potentiometers to the values shown in the patching diagram. Connect $x$ (the output of the inverter) to OUT X and connect your oscilloscope or other display system to RCA Out X on the back of THAT. If more channels are available on your visual display system, connect the $y$ and $z$ accordingly. Run the patch in OP mode and observe the image on the display system while connecting and disconnecting $I_{ext}$ to and from +1.

$$\dot{x} = -ax^3 + bx^2 + y - z + I_{ext}$$
$$\dot{y} = -dx^2 + c - y$$
$$\dot{z} = r(s(x - x_r) - z)$$
8.5 EULER SPIRAL

The Euler spiral is defined as a curve whose curvature increases linearly with its length. With the sign of its curvature following the sign of its length, the Euler spiral takes the shape of a double spiral with point-symmetry around the origin. Besides its visual appeal, the Euler spiral can be used to design smooth transitions between straight lines and circular arcs in two dimensions, which has practical value in several fields. An obvious and immediate application area is graphic design. The design of typography, in particular, requires numerous smooth transitions between straight and circular line segments. Another application area is road and rail traffic network design. To appreciate the benefits the Euler spiral offers in this area, imagine, as an alternative, the immediate tangential connection between straight and circular road segments. This would require motorists to change steering radii instantaneously, which is not only physically impossible but uncomfortable and dangerous to attempt. For similar reasons, the Euler spiral is used to generate movement paths in high-speed machinery and robotics. It has also been found to offer a good description of the curvature of rat whiskers.

To display the Euler spiral, a display system capable of visualizing two channels in x/y mode is required. Set the coefficient potentiometers 1 through 5 to the values shown in the patching diagram. Connect x (the output of the inverter shown in blue in the patching diagram) to OUT X and connect y (the output of the inverter shown in green in the patch diagram) to OUT Y. Connect your oscilloscope or other display system to RCA Out X and Y on the back of THAT. Set the display system to x/y mode and run the patch in REPF (repeat fast) mode. Vary the settings of the coefficient potentiometers 1 through 5 as well as the OP-Time potentiometer and observe the changing image on the display system.

\[
x(t) = \int_0^T \cos \left( \frac{t^2}{2} \right) \, dt
\]

\[
y(t) = \int_0^T \sin \left( \frac{t^2}{2} \right) \, dt
\]
How do the population sizes of hunter and prey species interact? An initial quantitative insight into this question was found in the early 20th century in the trading records of the Hudson's Bay Company, a Canadian-based fur trading business at the time. The company recorded the numbers of lynx and snowshoe hare pelts it bought from trappers, thereby creating an indicative historical documentation of the changing sizes of Canadian lynx and snowshoe hare populations. These records – the 1845 to 1935 data is shown on the right – reveal a curious pattern of periodic and temporally related increases and decreases in both population sizes.

As the lynx prey almost exclusively on the snowshoe hare, it was unsurprising to see the size of the hare population affected by the size of the lynx population. It seemed puzzling, however, that the size of the lynx population also seemed to be affected by the size of the hare population, leading some to ask: “Do hare eat lynx?” An explanation for these predator-prey population dynamics was proposed independently by Alfred J. Lotka in 1925 and Vito Volterra in 1926 in the form of the following two first-order differential equations:

\[
\begin{align*}
\frac{dh}{dt} &= (\alpha - \beta l) h \\
\frac{dl}{dt} &= (\delta h - \gamma) l
\end{align*}
\]

Variable \( h \) refers to the size of the hare population, while variable \( \alpha \) refers to the growth rate of the hare population (based on their consumption of plant life, which is assumed to be abundantly available in the environment). Variable \( l \) refers to the size of the lynx population, and variable \( \delta \) refers to the growth rate of the lynx population (based on their consumption of the changing hares population). The natural death rate of lynx is denoted by variable \( \gamma \), and the rate at which hare are killed by lynx is denoted by variable \( \beta \). As it turns out, hare do, of course, not eat lynx, but an excessively large lynx population will overhunt the hare population, thereby depleting the lynx's food source, which, in turn leads to a decimation of the lynx population.

To display the changing population sizes of both the predator and the prey species simultaneously, a visual display system capable of visualizing at least two channels is required. Set the coefficient potentiometers 1 through 6 to the values shown in the patch diagram on the right. Set THAT to OP mode and your visual display system to “roll mode.” To explore the dynamics of both interacting populations further, change the settings of coefficient potentiometers 1 through 6 interactively as the patch runs. If possible, also try visualizing the output in x/y mode on your display system. The result is a so-called phase-space plot for the modeled predator prey relationship. This patch can be used to model competitive relationships beyond ecological systems, for example in economic systems and communicable diseases.
9. FREQUENTLY ASKED QUESTIONS

Why do the plugs not go fully into the patch panel?
The 2 mm plug cables were originally designed to plug entirely into a corresponding type of gold-plated socket. One of these sockets plus mounting costs about USD 1.00, which would add up significantly for the 186 plug positions on THAT’s patch panel. We saved most of this cost by using an extra-thick top circuit board with appropriately-sized, gold-plated through-holes. Since the length of the plugs is greater than the thickness of the circuit board, we placed stop-limits below the patch field to ensure that the small, contact-assuring springs halfway along the length of each plug make reliable contact.

Is THAT a general-purpose computer?
Yes and no. The term general-purpose computer is commonly used to describe digital stored-program computers that can execute arbitrary algorithms. While THAT does not belong in this category, it is a general-purpose analog computer in that it can solve any (set of) differential equation(s) within the means of its computing elements. By connecting multiple THATs in master/minion chains, it is possible to implement arbitrarily large analog computer patches involving any number of computing elements.

Is there a template to draw and share THAT patches?
Yes. You can download it from the THAT website at this URL: https://the-analog-thing.org/THAT_template.pdf

How precise is THAT compared to a digital computer?
THAT is precise to about two positions after the decimal point, relative to its Machine Unit (±1). Comparing the precision of analog and digital computers is a bit like comparing apples and oranges. Analog computers usually handle quantities based on measuring only (“What is your body height?”). Digital computers, however, also handle quantities based on counting (“How many siblings do you have?”), which requires strict numerical precision. Consider this: A bank clerk getting the third decimal place of an interest rate wrong commits a severe error, while a tailor being off by a few micrometers when taking a client’s measurements has no such problem. Furthermore, numerical digital computing involves rounding, and hence rounding errors, which can add up quickly in iterative loops. Analog computers do not operate numerically and do not round. In this sense, the great precision of today’s digital computers helps minimize a problem that is specific primarily to digital computing. Representing quantities as continuous voltages, THAT does not suffer from many issues that are inherent to binary value representations. While analog computer solutions can be affected by noise and instabilities, the precision of THAT is perfectly appropriate for most analog computer applications.

10. FURTHER READING

Books

Online
- THE ANALOG THING: https://the-analog-thing.org
- THE ANALOG THING online documentation: https://the-analog-thing.org/docs/dirhtml
- Analog Paradigm: https://analogparadigm.com
- anabrid: https://www.anabrid.com