x-pendula

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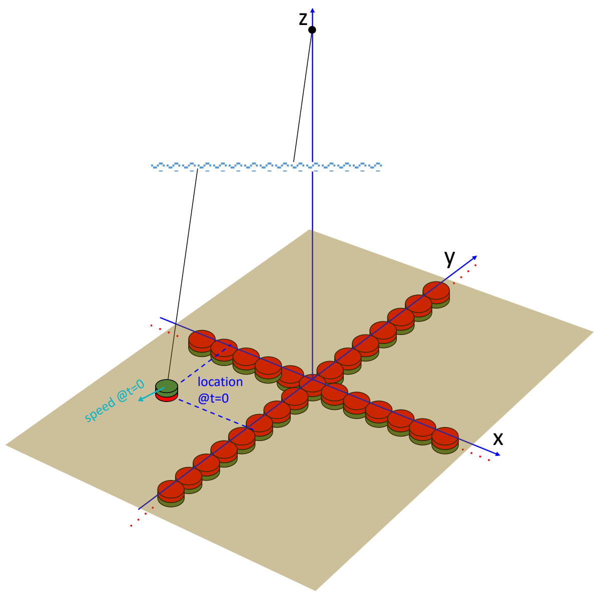
Sept. 2024

Goal

Example of a chaotic pendulum by using a single THAT and an oscilloscope in xy operation mode. This application shall correspond to a roulette table with 4 steady states and a ball with adjustable starting speed and starting position.

Setup

The setup is generated for enabling the implementation of the differential equation with a single THAT.



The ball is represented by an infinite small magnet at the end of a long fiber. The fiber is fixed on (0;0;→∞); the magnet hovers shortly above the xy-plane.

At the xy-axels infinite small magnets are mounted.

The ball hat 2 initialized conditions: location and speed @t=0.

Equation

The balanced forces can be described easily, assuming ball and magnets are on z=0:

normalized different equation

$$\underbrace{\underbrace{\begin{pmatrix}\ddot{x}\\ \ddot{y}\end{pmatrix}}_{\substack{acceleration\\ \ddot{\vec{r}}}}}_{\text{"inertia"}} = \underbrace{-\begin{pmatrix}\alpha_x\\ \alpha_y\end{pmatrix}\cdot\underbrace{\begin{pmatrix}\dot{x}\\ \dot{y}\end{pmatrix}}_{\substack{speed\\ \dot{\vec{r}}}}}_{\substack{adjustable\\ \text{"friction"}\\ \to\alpha}} + \underbrace{\begin{pmatrix}\beta_x\\ \beta_y\end{pmatrix}\cdot\begin{pmatrix}1/x\\ 1/y\end{pmatrix}}_{\substack{adjustable\\ \text{"force}\\ of\\ \text{repulsion"}\\ \to\beta}} \underbrace{-\begin{pmatrix}\gamma_x\\ \gamma_y\end{pmatrix}\cdot\underbrace{\begin{pmatrix}x\\ y\end{pmatrix}}_{\substack{position\\ \vec{r}}}}_{\substack{adjustable\\ restoring\\ \text{"force}\\ of\\ the\\ \text{pendula"}\\ \to\gamma}}$$

Hint: The symbol $\cdot$ shall represent a standard multiplication only, not a scalar product.

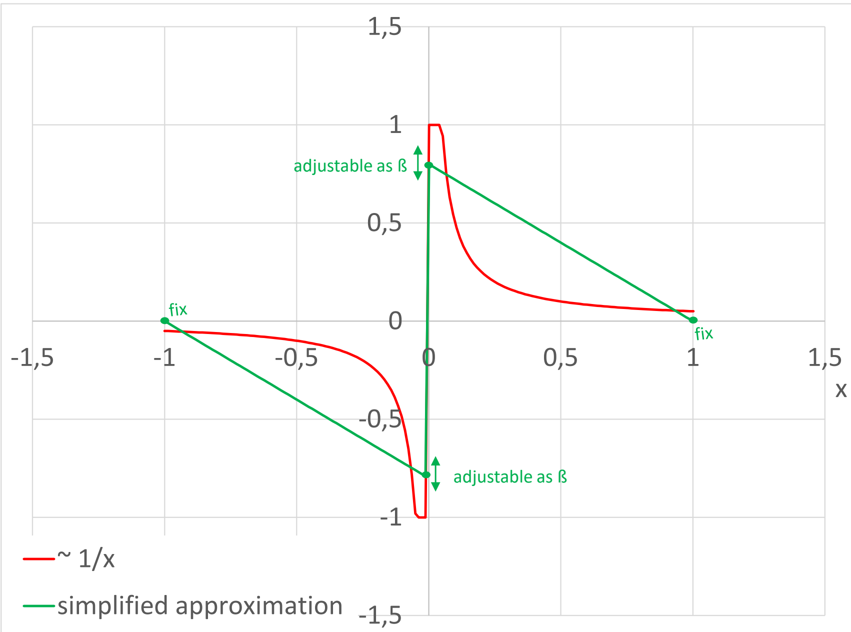
Unfortunately we get at x=0 or at y=0: division by zero. Putting the ball to z=h avoids division by zero:

normalized different equation $$\begin{pmatrix}\ddot{x}\\ \ddot{y}\end{pmatrix}=-\begin{pmatrix}\alpha_x\\ \alpha_y\end{pmatrix}\cdot\begin{pmatrix}\dot{x}\\ \dot{y}\end{pmatrix}+\begin{pmatrix}\beta_x\\ \beta_y\end{pmatrix}\cdot\begin{pmatrix}x/_{x^2+h^2}\\ y/_{y^2+h^2}\end{pmatrix}-\begin{pmatrix}\gamma_x\\ \gamma_y\end{pmatrix}\cdot\begin{pmatrix}x\\ y\end{pmatrix}$$

Now it's easy to be implemented, however one THAT does not offer enough calculation components (4 multipliers are required here, 2 are offered by THAT).

Hint:
1/x cannot be implemented for x $\in[-1;+1]$ in THAT (the denominator has to be >0 always).

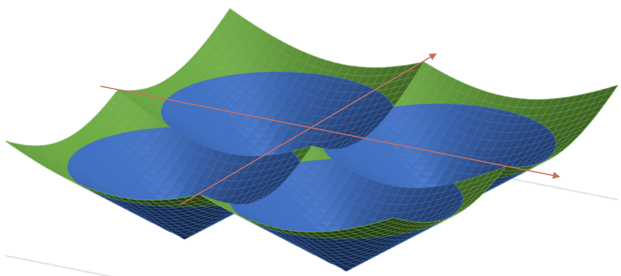
Let's simplify more just to get it implementable.



The red curve shows an example of $\frac{\beta_x}{x}$

The normalized result is limited to ±1 (property of a THAT component).

The green curve shows a simplified straight forward approach "similar" to the red curve.

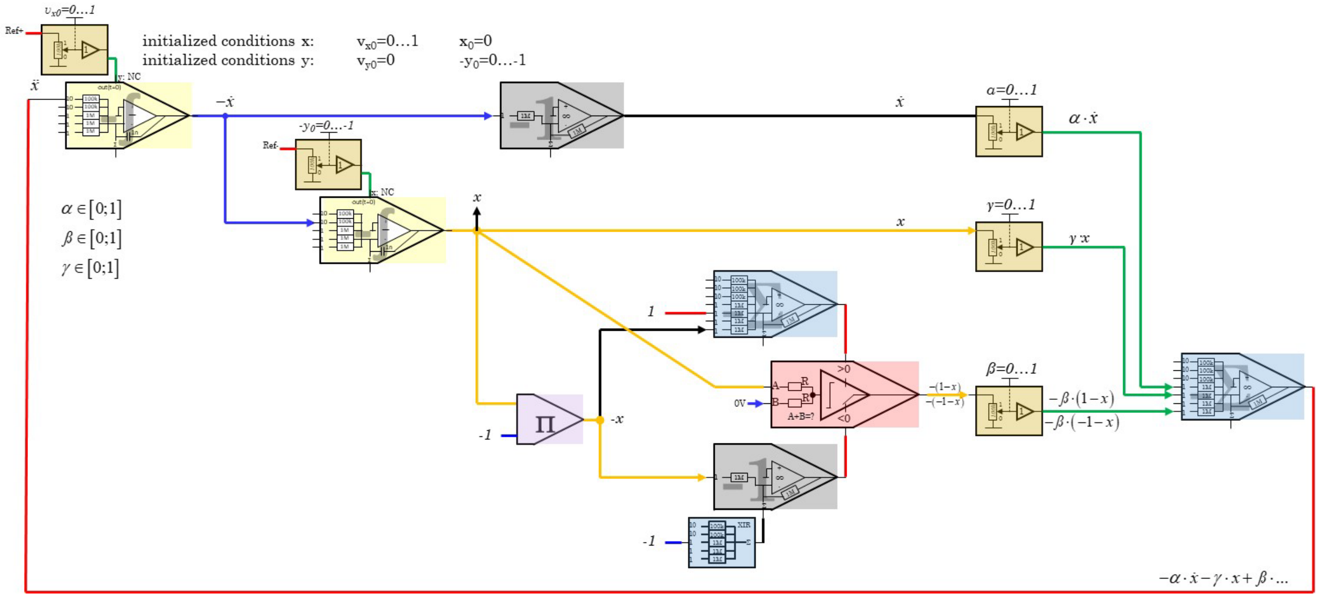


Based on the simplification above, the 3D-map of the static forces shows the steady state operating points clearly: one by quadrant.
It can be expected that the 1/x and the approximated behavior are going to produce more or less similar results.

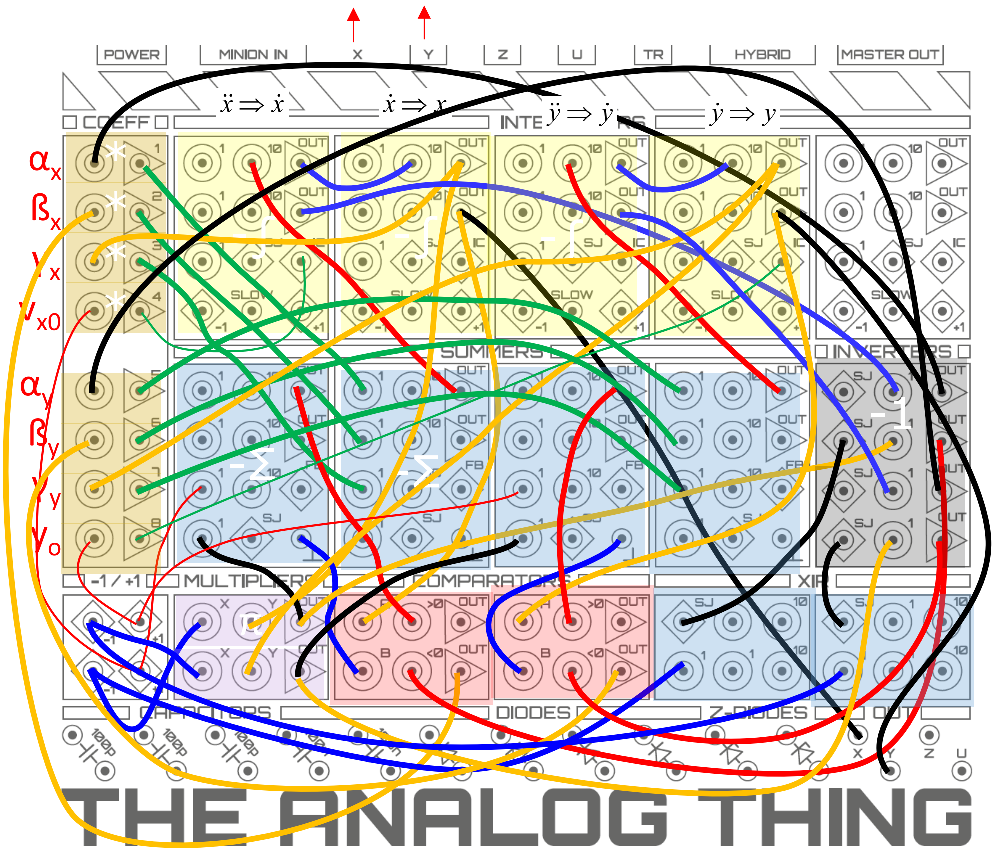
finally normalized different equation $$\underbrace{\begin{pmatrix}\ddot{x}\\ \ddot{y}\end{pmatrix}}_{\ddot{r}}=-\begin{pmatrix}\alpha_x\\ \alpha_y\end{pmatrix}\cdot\underbrace{\begin{pmatrix}\dot{x}\\ \dot{y}\end{pmatrix}}_{\dot{r}}+\begin{pmatrix}\beta_x\\ \beta_y\end{pmatrix}\cdot\begin{pmatrix}\begin{cases}1-x & for\ x>0\\ -1-x & for\ x<0\end{cases}\\ \begin{cases}1-y & for\ y>0\\ -1-y & for\ y<0\end{cases}\end{pmatrix}-\begin{pmatrix}\gamma_x\\ \gamma_y\end{pmatrix}\cdot\underbrace{\begin{pmatrix}x\\ y\end{pmatrix}}_{r}$$

This equation is designed for being implemented in THAT: as many components as possible shall be used (further simplifications of the equation are possible).

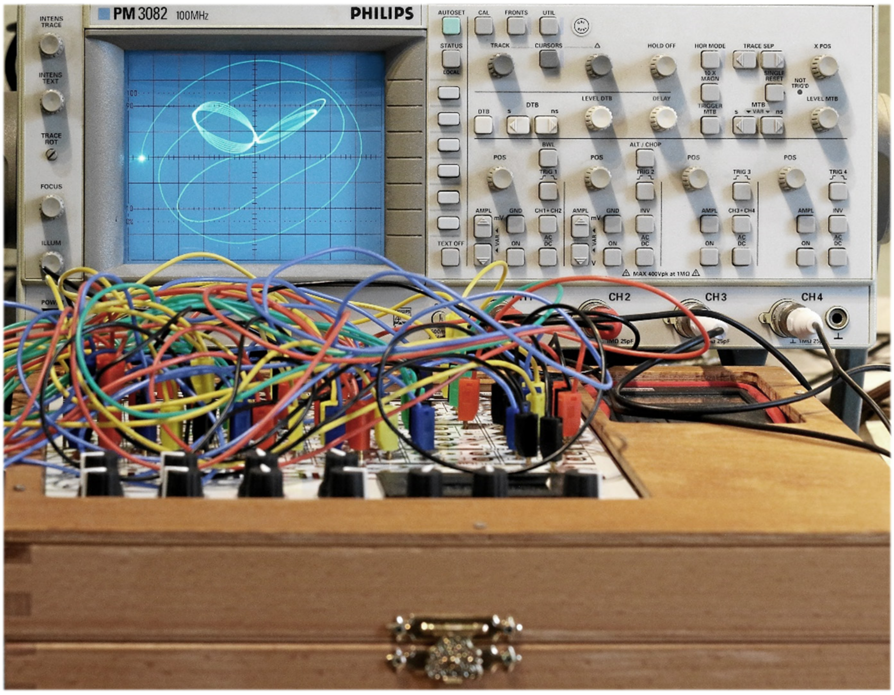
Implementation



The block chart has to be implemented twice: for x and for y.
Hint: I'm a hard core electronics engineer – I prefer an "electronics" block chart.



Colored wiring according the block chart above.

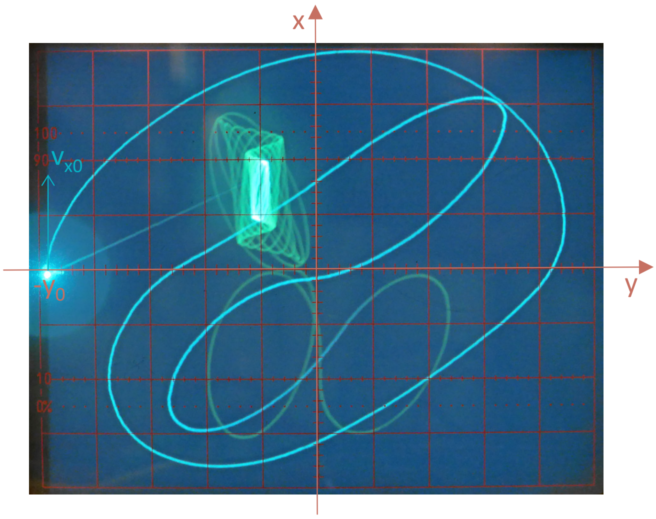


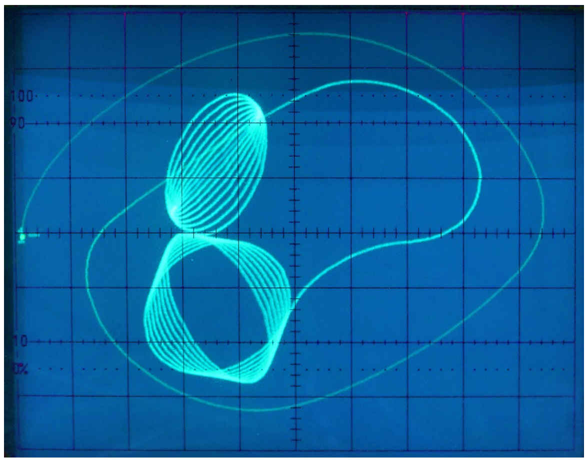
Real implementation in operation: the chaotic behaviour can be seen easily.

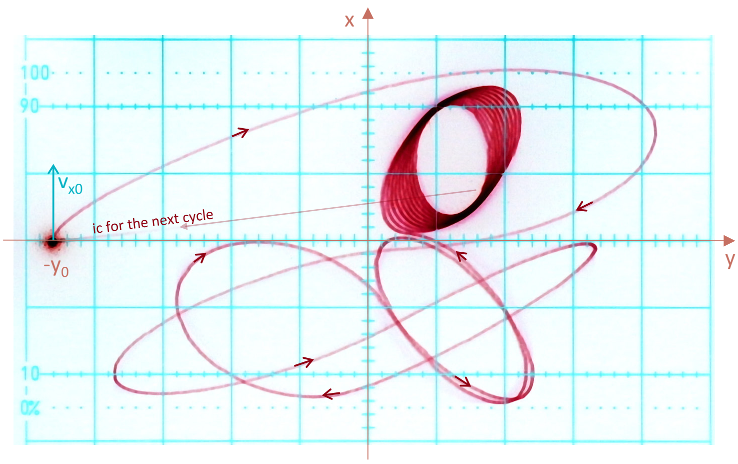
Hint: The oscilloscope shows the y-axel horizontally and the x-axel vertically.

Hint: I put THAT together with a litle single channel oscilloscope into a wooden case.

Results







Three examples with various adjustments.

Adjustable parameter:
initialized conditions $-y_0$ and v_{x0}
a, ß and γ for the x-axel
a, ß and γ for the y-axel