x-pendulum

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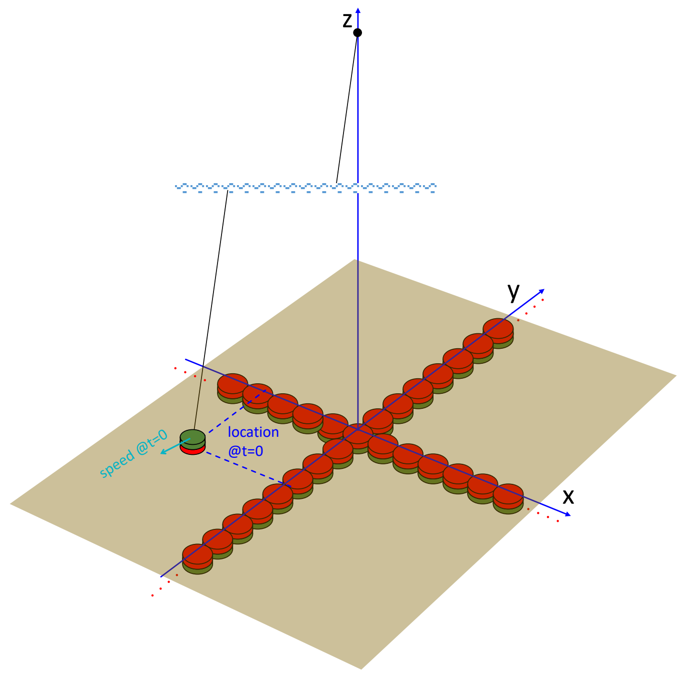
Goal

Example of a chaotic pendulum by using a single THAT and an oscilloscope in x-y-operation mode. This application shall correspond to a roulette table with 4 steady states and a ball with adjustable starting speed and starting position.

Setup

The setup is generated for enabling the implementation of the differential equation with a single THAT.



The ball is represented by an infinite small magnet at the end of a long fiber. The fiber is fixed on (0;0;→∞); the magnet hovers shortly above the x-y-plane.

At the x-y-axes infinite small magnets are mounted.

The ball hat 2 initialized conditions: location and speed @t=0.

Equation

The balanced forces can be described easily, assuming ball and magnets are on z=0:

normalized different equation

$$\underbrace{\underbrace{\begin{pmatrix}\ddot{x}\\ \ddot{y}\end{pmatrix}}_{\substack{acceleration\\ \ddot{\vec{r}}}}}_{\text{"inertia"}} = -\underbrace{\begin{bmatrix}\alpha_x & 0\\ 0 & \alpha_y\end{bmatrix}\cdot\underbrace{\begin{pmatrix}\dot{x}\\ \dot{y}\end{pmatrix}}_{\substack{speed\\ \dot{\vec{r}}}}}_{\substack{adjustable\\ \text{"friction"}\\ \to\alpha}} + \underbrace{\begin{bmatrix}\beta_x & 0\\ 0 & \beta_y\end{bmatrix}\cdot\begin{pmatrix}1/x\\ 1/y\end{pmatrix}}_{\substack{adjustable\\ \text{"force}\\ of\\ \text{repulsion"}\\ \to\beta}} - \underbrace{\begin{bmatrix}\gamma_x & 0\\ 0 & \gamma_y\end{bmatrix}\cdot\underbrace{\begin{pmatrix}x\\ y\end{pmatrix}}_{\substack{position\\ \vec{r}}}}_{\substack{adjustable\\ restoring\\ \text{"force}\\ of\\ the\\ \text{pendula"}\\ \to\gamma}}$$

Unfortunately we get at x=0 or at y=0: division by zero. Putting the ball to z=h avoids division by zero:

normalized different equation

$$\begin{pmatrix}\ddot{x}\\ \ddot{y}\end{pmatrix} = -\begin{bmatrix}\alpha_x & 0\\ 0 & \alpha_y\end{bmatrix}\cdot\begin{pmatrix}\dot{x}\\ \dot{y}\end{pmatrix} + \begin{bmatrix}\beta_x & 0\\ 0 & \beta_y\end{bmatrix}\cdot\begin{pmatrix} x \big/ x^2+h^2\\ y \big/ y^2+h^2\end{pmatrix} - \begin{bmatrix}\gamma_x & 0\\ 0 & \gamma_y\end{bmatrix}\cdot\begin{pmatrix}x\\ y\end{pmatrix}$$

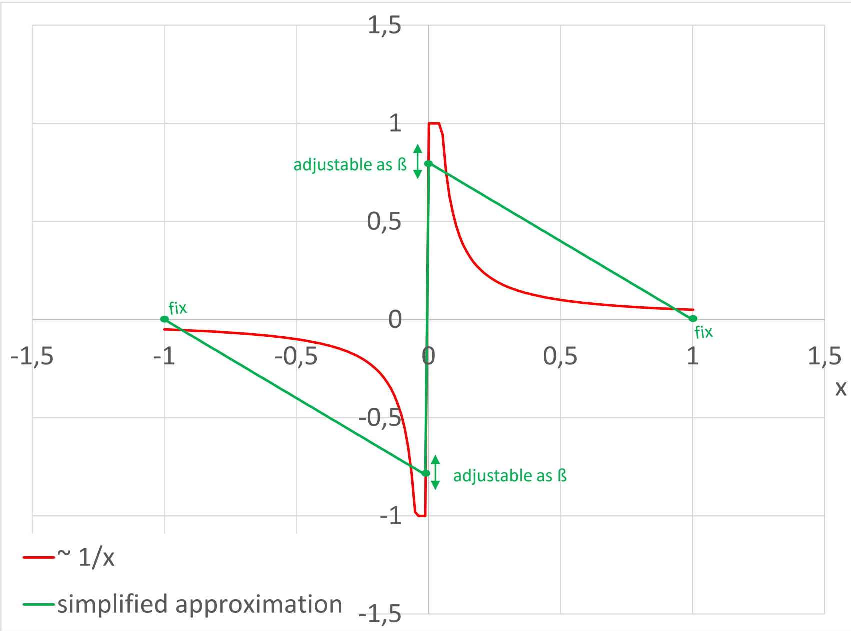
(skipping the z-axis)

Now it's easy to be implemented, however one THAT does not offer enough calculation components (4 multipliers are required here, 2 are offered by THAT).

Hint:

$^1/_x$ cannot be implemented for x $\in\left[-1;+1\right]$ in THAT (the denominator has to be >0 always).

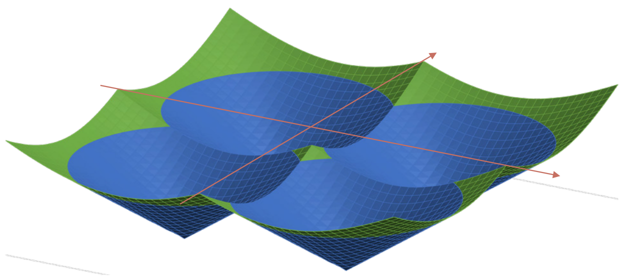
Let's simplify more just to get it implementable.



The red curve shows an example of $\dfrac{\beta_x}{x}$

The normalized result is limited to ±1 (property of a THAT component).

The green curve shows a simplified adjustable straight forward approach "similar" to the red curve.



Based on the simplification above, the 3D-map of the static forces shows the steady state operating points clearly: one by quadrant.

It can be expected that the 1/x and the approximated behavior are going to produce more or less similar results.

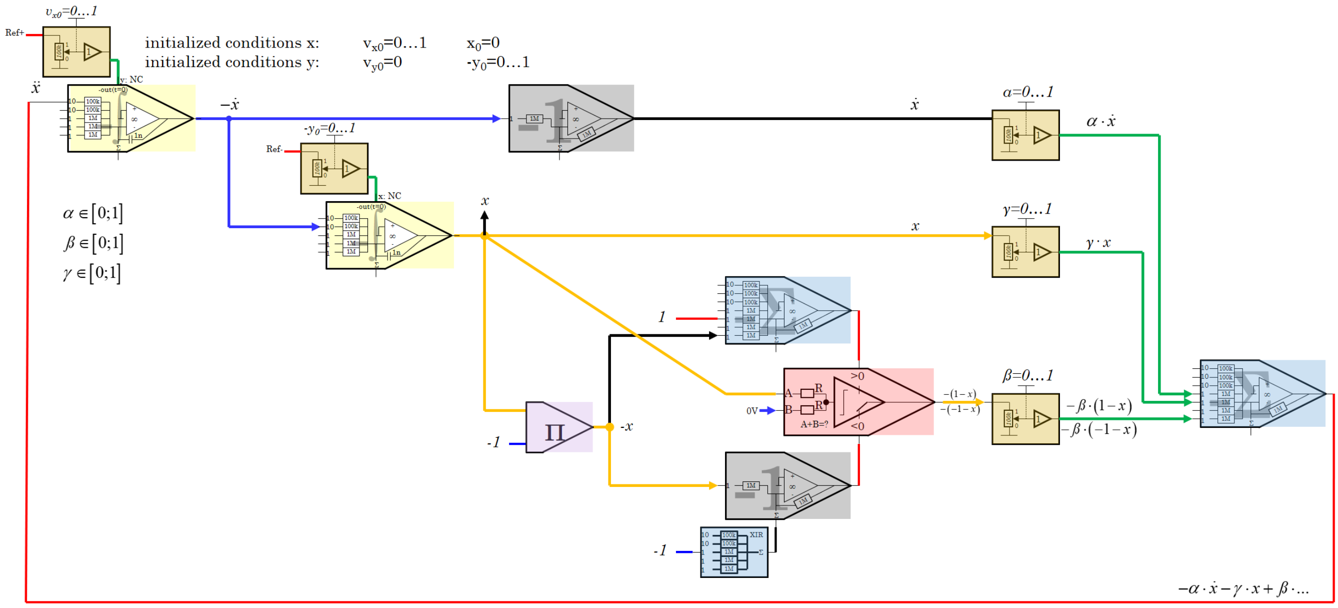
finally normalized different equation

$$\underbrace{\begin{pmatrix}\ddot{x}\\ \ddot{y}\end{pmatrix}}_{\ddot{\vec{r}}} = -\begin{bmatrix}\alpha_x & 0\\ 0 & \alpha_y\end{bmatrix}\cdot\underbrace{\begin{pmatrix}\dot{x}\\ \dot{y}\end{pmatrix}}_{\dot{\vec{r}}} + \begin{bmatrix}\beta_x & 0\\ 0 & \beta_y\end{bmatrix}\cdot\begin{pmatrix}\begin{cases}1-x & for\ x>0\\ -1-x & for\ x<0\end{cases}\\ \begin{cases}1-y & for\ y>0\\ -1-y & for\ y<0\end{cases}\end{pmatrix} - \begin{bmatrix}\gamma_x & 0\\ 0 & \gamma_y\end{bmatrix}\cdot\underbrace{\begin{pmatrix}x\\ y\end{pmatrix}}_{\vec{r}}$$

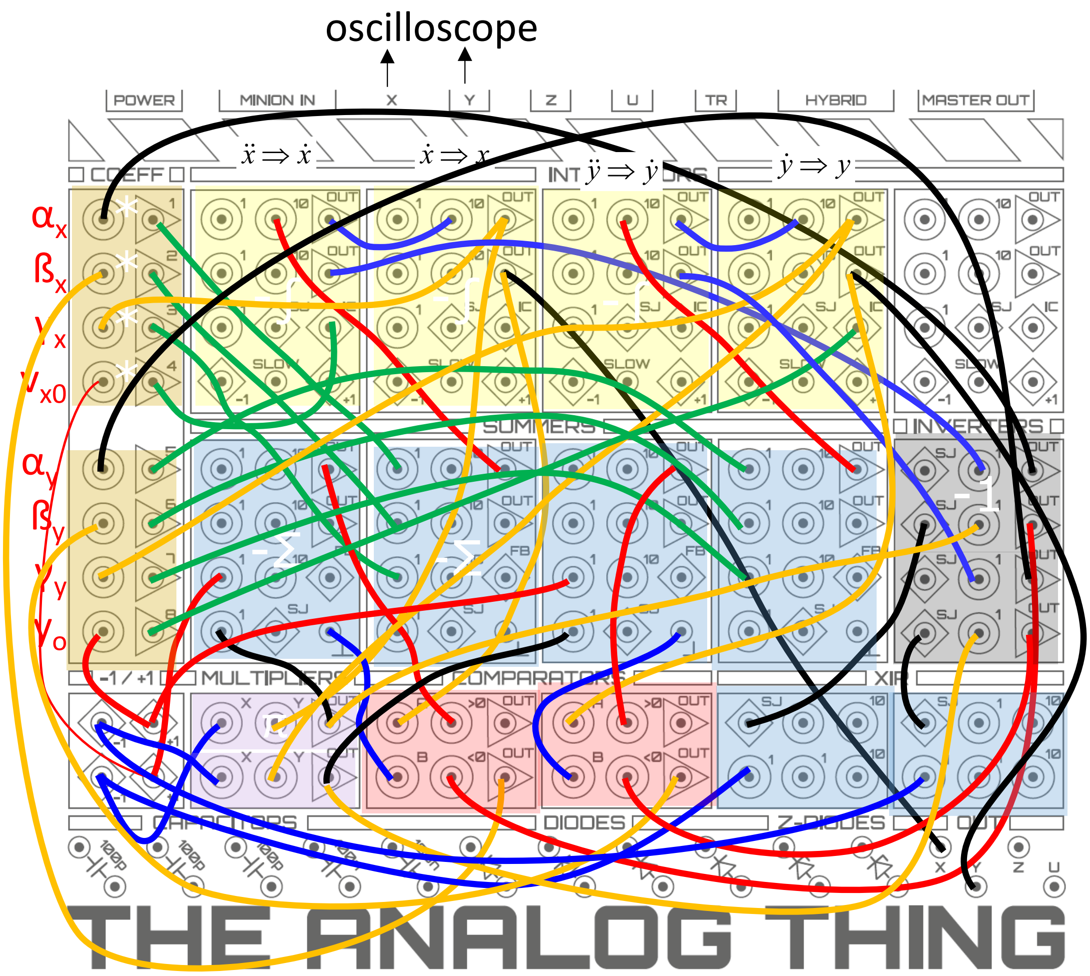
Hint: the behaviour at x=0 is technically irrelevant. The comparator in THAT has an "unknown" offset voltage ≠ 0V.

This equation is designed for being implemented in THAT: as many components as possible shall be used (further simplifications of the equation are possible).

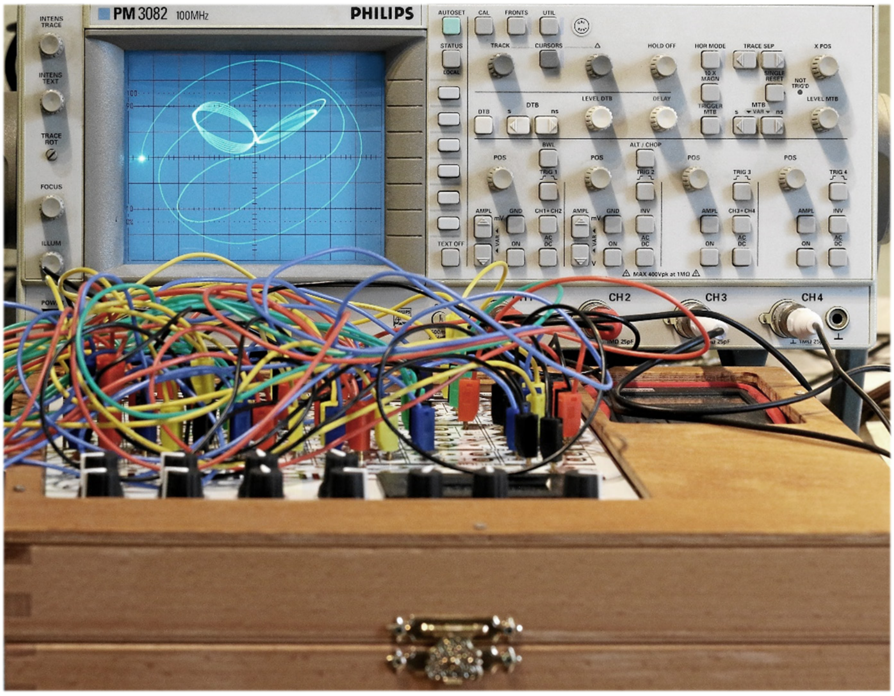
Implementation



The block chart has to be implemented twice: for x and for y.
Hint: It's an "electronic" block chart for supporting the view on the "internal operating of THAT".



Colored wiring according the block chart above.

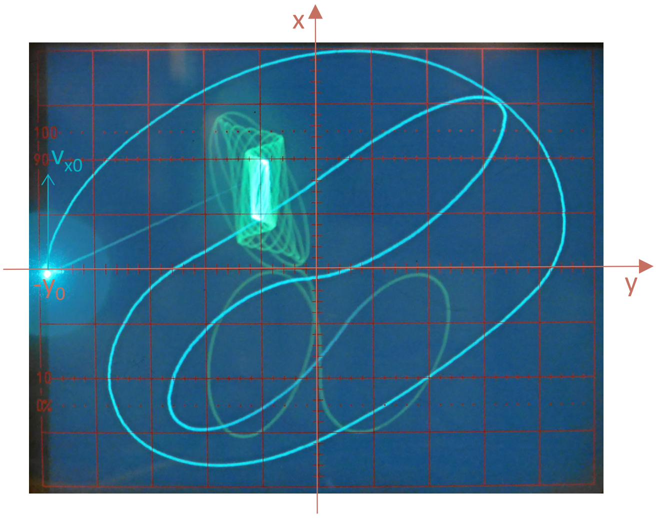


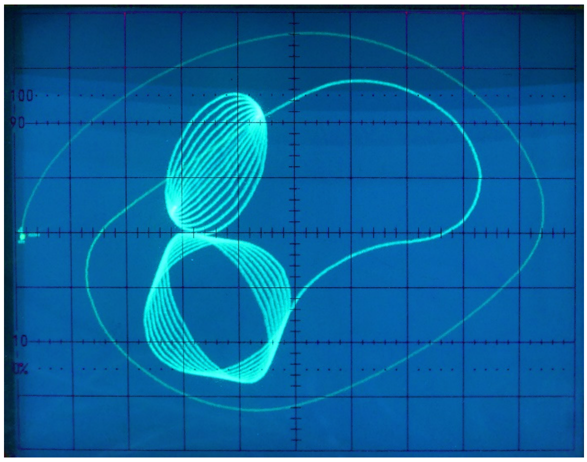
Real implementation in operation: the chaotic behaviour can be seen easily.

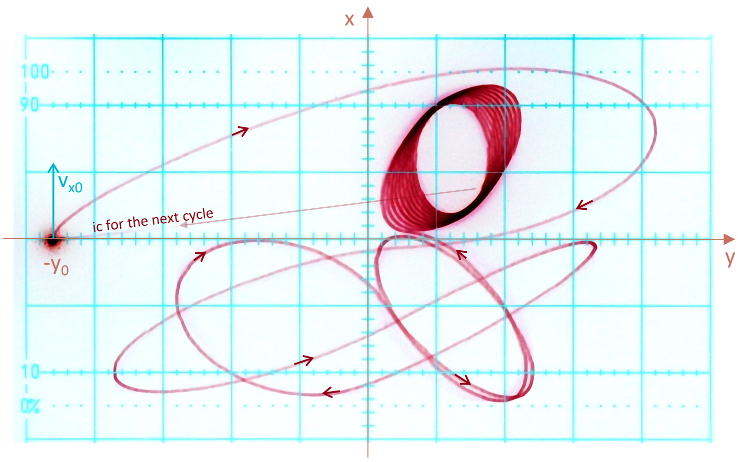
1st Hint: The oscilloscope shows the y-axis horizontally and the x-axis vertically.

2nd Hint: THAT is mounted together with a litle single channel oscilloscope into a wooden case here.

Results







Three examples with various adjustments.

Adjustable parameter:
initialized conditions $-y_0$ and v_{x0}
a, ß and γ for the x-axis
a, ß and γ for the y-axis